Example of exam questions in Time Series Econometrics

Most questions require that you tell a story about how to solve a given problem, many relates back to the labs. Notice that the lab instructions, and other material distributed in labs, contain detailed discussions about the econometric problems in lab. They are not simply instructions about estimating a problem and present a number.

The questions cover a number of courses and not all courses have been taught in the same way. Thus, there can be some questions goes a little besides what is covered in your course.

QUESTION 1. Unit root tests

Under assumptions of rational expectations, some theories predict that private consumption will be a random walk variable. Explain and show how you can test for the random walk hypothesis for private consumption. Suppose that consumption ($x_t$) is driven by the following process,

$$x_t = x_{t-1} + \epsilon_t,$$

where $\epsilon_t$ is NID(0, $\sigma^2$). Notice that there are two hypothesis involved, first that the coefficient on the lagged variable is unity, and that the residual ($\epsilon_t$) in the above equation is white noise. In the answer try to explain in detail why the t-statistic from an OLS estimation of the above model might lead you wrong.

QUESTION 2. Co-integration

Private consumption ($c_t$) is often modeled as a function of income ($y_t$) and wealth ($W_t$). All three variables can generally be characterized as integrated variables. Explain why you cannot go ahead and estimate this model by OLS. Explain in detail how one can estimate an error correction model, from the data series above, where statistical inference based on standard distributions is possible.

QUESTION 3. TIME Series Models

a) Which are the different steps in the Box-Jenkins approach for time series modeling? (3 marks)

b) How do you test for white noise and on which basis do you select a final model? (3 marks)

c) Assume the following AR(1) model,

$$x_t = px_{t-1} + \epsilon_t,$$

where $\epsilon_t$~NID(0, $\sigma^2$), explain the differences of performing inference on the estimated $p$, when $|p| < 1.0$ and when $p=1.0$. (4 marks)
QUESTION 4. UNIT ROOT TESTS

a) What are the characteristics of a variable integrated of order one, compared with a stationary variable? (3 marks)

b) Assume that \( x_t \) is a typical macroeconomic time series, like the nominal money stock, explain how you test if \( x_t \) is integrated of order one. (5 marks)

QUESTION 5. ARIMA MODELS

a) Define the autocorrelation function (ACF) and the partial autocorrelation function (PACF).

b) How do you test for the significance of the estimated AFCs and PACFs. Explain and show carefully why t-values cannot be used to test the significance of estimated PACFs.

c) Show the duality between MA and AR models.

d) What is meant by weak stationarity, and covariance stationarity?

QUESTION 7. UNIT ROOT TESTS

a) What are the characteristics of a variable integrated of order one, compared with a stationary variable?

b) In a test of I(1), against I(0), discuss the possible alternative hypothesis that might exist. Indicate how you can test for the alternatives.

c) In the Dickey-Fuller model \( \Delta x_t = \beta x_{t-1} + \epsilon_t \), where the true value of \( \beta \) is equal to zero. Explain why you cannot use a standard t-test to examine if \( \beta \) equals zero.

QUESTION 8. Cointegration

a) Assume that you have the following time series: \( x_t \sim I(1), \ y_t \sim I(1), \) and \( z_t \sim I(2) \). How would you characterize a stable long-run relationship between these variables. Discuss how these variables can be combined to form possible stable long-run relationships.
b) Suppose that you test for cointegration between \( x_t \sim I(1) \) and, \( y_t \sim I(1) \) using the two-step procedure. Describe how you would perform this test. Explain why the t-statistics is useless for testing the null of no relationship between the variables.

c) Discuss the limitations of the Engle and Granger’s two-step procedure.

d) Suppose that you have the following \( p \)-dimensional multivariate process \( \{y_t\} \), with the following VAR representation,

\[
y_t = \sum_{i=1}^{k} A_i y_{t-i} + \Phi D_t + \epsilon_t
\]

where \( A_i \) is a matrix of coefficients associated with lag \( i \), \( D_t \) is a vector of deterministic variables, and \( \epsilon_t \sim NID(0, \Omega) \). Rewrite this VAR into a Vector Error Correction Model (VECM).

**Question 9. (10 points)**

This type question can be varied in different ways, of course. The principle is the same identify the problems and the errors that has been made. Explain how and why things should be improved. At higher levels you can explain things even better if you use density functions.

We have the following estimated model,

\[
\Delta \ln \text{Cons}_t = B_0 + B_1 \Delta \ln \text{GDP}_t + B_2 \Delta \ln \text{CPI}_t + B_3 \Delta \text{R60c}_t + \epsilon_t
\]

where \( \Delta \) is the first difference operator, GDP is gross domestic product, CPI is consumer price index, R60c is the three-month treasury bill rate.

The present sample is: 1969 (2) to 1994 (1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>HCSE</th>
</tr>
</thead>
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<tr>
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<td>0.00338</td>
<td>5.001</td>
<td>0.004597</td>
</tr>
<tr>
<td>( \Delta \ln \text{GDP}_t )</td>
<td>0.36086</td>
<td>0.09173</td>
<td>3.934</td>
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</tr>
<tr>
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<td>0.25917</td>
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<td>1.613</td>
<td>0.241880</td>
</tr>
<tr>
<td>( \Delta \text{R60c}_t )</td>
<td>0.00131</td>
<td>0.00103</td>
<td>1.269</td>
<td>0.001131</td>
</tr>
</tbody>
</table>

\( R^2 = 0.174366 \)  \( F(3,96) = 6.7581 [0.0003] \)  \( DW = 2.35 \)

\( RSS = 0.02298197499 \) for 4 variables and 100 observations

AR 1-2 \( F(2, 94) = 1.8497 \) [0.1630]
ARCH 4 \( F(4, 88) = 2.6102 [0.0408] \)*
Normality Chi^2+ (2)= 4.4048 [0.1105]
RESET \( F(1, 95) = 0.94744 [0.3328] \)

Comment on the estimated model and its results. Can the estimation be improved?
Question(s)

a) What are the differences between a martingale and a random walk?

b) What is an innovation process?

c) Why is finance theory saying that asset prices should be martingales and not random walks?
Question 1. A real life estimation that might have gone wrong

Look at the following estimated time series model describing the demand for electricity in Namibia.¹

\[
\ln Q_{el,t} = a_0 + a_1 \ln P_{el,t} + a_2 \ln P_{coal,t} + a_3 \ln DS\_GDP_t + e_{1,t}
\]

Where GDP has been "deseasonalised" as

\[
GDP_t = b_0 + b_1 \text{TIME} + b_2 DRY_t + e_{2,t} \quad \text{and} \quad e_{2,t} = DS\_GDP_t
\]

Where,

- \( Q_{el,t} \) is demand for electricity.
- \( \text{Time} \) is a deterministic time trend.
- \( \text{DRY}_t \) is a dummy variable that is set to unity in periods of no drought and to zero when there was a drought. We do not have further information about \( \text{DRY} \), but we can conclude that it is a stationary variable; periods of drought are followed by periods of no drought.
- The demand for electricity (\( Q_t \)) is increasing over time, but it might be a stationary series. It is growing during the 80s, but is quite stable during the 90s.
- The price of electricity (\( P_{el} \)) is constantly falling during the 1980 to 1994. From around 13 cents/kWh to around 6 cents/kWh.
- The price of coal (\( P_{coal} \)) is falling during the period. Coal prices peaked during the second oil crisis and falls thereafter.
- This model has been estimated over the period 1980 - 1996 using yearly data, but something might have gone terribly wrong, can these results be trusted?

The following results are presented, based on OLS:

**Electricity demand using GDP,**

\[
\ln Q_{el,t} = a_0 - 0.8628 \ln P_{el,t} - 0.0042 \ln P_{coal,t} - 0.5118 \ln GDP_t \quad R^2 = 78.9
\]

(t-ratios) (-4.67) (-0.02) (-1.32)

**Estimating DS\_GDP**

\[
GDP_t = 4823 + 86.1 \text{TIME} + 355 \text{DRY}_t \quad R^2 = 74.8
\]

¹ The model and the results are taken from an article published in Journal of Development Alternatives and Area Studies no. 1&2/2001.
Estimating electricity demand using DS GDP

\[ \ln Q_{el,t} = a_0 - 0.342 \ln P_{el,t} + 0.191 \ln P_{coal,t} + 0.891 \ln GDP_t \quad R^2 = 81.9 \]

(t-ratios) \(-1.15\) \(0.807\) \(1.27\)

No other information, than \(R^2\) and t-ratios, are given in the paper.

Here, the main problem is not the short sample, but something much more fundamental. Write a short essay, based on the course material. Explain what are the problems, why are they problems, and how should they be solved?

To your help, besides the information above, you have the following graphs produced from official data from Namibia. In what way can the graphs and statistics below help you?

Data for Namibia.
Question 2. The Principle of Duality (10 marks)

Explain the principle of duality and its practical implications for time series modeling.

Question 3. Unit Root Tests (15 Marks)

a) What are the characteristics of an I(1) variable, compared with an I(0) variable?

b) In a test of I(1), against I(0), discuss the possible alternative hypothesis that might exist. Indicate how you can test for the alternatives in as many ways as possible.

c) In the model $\Delta x_t = \beta x_{t-1} + \varepsilon_t$, where the true value of $\beta$ is equal to zero. Explain why you cannot use a standard $t$-test to examine if $\beta$ equals zero.

Question 4. Error correction (15 Marks)

a) What is the long-run solution to the following equation?

$$y_t = 0.7 + 0.55y_{t-1} + 0.35y_{t-2} + 0.05y_{t-3} + 0.2x_t + 0.1x_{t-1} + 0.05x_{t-2} + \varepsilon_t$$

b) What is the economic interpretation of the long-run solution?

c) Explain briefly how we, using mathematical methods, can judge how $\{y_t\}$ is evolving over time? (Explosive, stationary, etc.) Is the series above stationary?

d) Rewrite the equation as a single error correction model. Explain briefly the advantage of the error correction solution.

e) Rewrite the system as a general VAR (2) model, and thereafter as a general VECM process. Where and how do you find the information about the long run?

Question 5

a) Show the link between a VAR and structural economic model?
b) Explain briefly what you can learn from variance decomposition and impulse responses?

c) Explain briefly why orthogonalization is necessary and how it can be achieved.
Question 1. 25 marks

a) What is meant by weak stationarity?

b) Discuss the definition and interpretation of an integrated series.

c) Are integrated series stationary? Is an ARMA series stationary? Is a random walk with a drift stationary? Explain why, why not?

Question 2. 25 Marks

Consider the ADL model in the three variables $Y_t, X_{1t},$ and $X_{2t}$:

\[ Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \epsilon_t \]

2a) Reformulate and interpret this model into an error correction model. What is the long-run solution?

2b) Suppose that $Y_t, X_{1t}$ and $X_{2t}$ can be described as random walks. Discuss the consequences of estimating and testing the significance of $B_0, B_1$ and $B_2$ in this case.

2c) Describe how you can solve the problem in question 2b, if you now that the variables are random walks. Suggest ways of approaching the problem.

2d) Also, in your answer try to hint at the difference of a regression where the X:s are random walks and were they can be assumed as stationary and "fixed in repeated samples"

Question 3.

In time series modelling there is something called duality. Explain the principle of duality. What is the practical consequence of duality for applied time series modelling?

Question 4.

Derive the OLS estimator for the model

\[ y_t = B x_t + \epsilon_t \]

where it assumed that $y_t$ and $x_t$ are adjusted for their means.
What are the requirements of this OLS model for understanding the estimated parameter as a statistical estimated as an economic variable?

Use the results above to discuss the possible bias in the estimate of B.

**Question 5.**

You are given the following process

\[ y_t = 0.3 + 0.7 y_{t-1} - 0.2 y_{t-2} + 0.5 y_{t-3} - 0.4 y_{t-4} + e_t, \]

where \( e_t \) is white noise.

a) What is the long run solution of the model?

b) How can you mathematically determine the future time path of this process?

c) Define the PACF and the ACF functions? And, what will they look like for this process?

d) How can you test if \( e_t \) really is white noise empirically?

**Question 1.**

a) What is meant by weak stationarity?

b) Is an ARMA(\( p, q \)) series stationary? Is a random walk with a drift stationary? Explain why, why not?

c) What is meant by a GARCH process and GARCH in mean?

**Question 2.**

a) Rewrite the following VAR as a Vector Error Correction Model,

\[ y_t = \pi_{11} y_{t-1} + \pi_{12} y_{t-2} + \pi_{13} x_{t-1} + \pi_{14} x_{t-2} + e_{1,t} \]
\[ x_t = \pi_{21} y_{t-1} + \pi_{22} y_{t-2} + \pi_{23} x_{t-1} + \pi_{24} x_{t-2} + \epsilon_{2,t} \]

b) Discuss the implication of cointegration in this model.

**Question 3.** 10 marks

Suppose that \( \{x_t\} \) is a time series process.

a) Explain how you can test if \( x_t \) is an integrated process?

b) Why can you not test for integration using ordinary t-tests, F-tests, etc.? (Explain in some detail)

**Question 4.** 10 marks

Suppose you have the following process \( y_t \sim ARIMA(2,2,0) \).

What is \( y_t \) for a type of process? Explain the use of the PACF and the ACF to identify the process. Show graphs, write down definitions.

**Question 5.** 10 marks

Define the terms and explain the differences, and perhaps similarities, between Markow processes, Martingales and Wiener processes (or Brownian motions).

**Question 6.** 10 marks

In the following model,

\[ y_t = \beta_1 y_{t-1} + \epsilon_t, \]

the residual is specified as \( \epsilon_t = \rho \epsilon_{t-1} + \nu_t \) where \( \nu_t \sim NID(0, \sigma^2) \). The coefficient \( \beta_1 \) is less than unity in absolute terms.

a) How can check if the residual is white noise in a model like this?

b) What happens if you try to OLS to estimate \( \hat{\beta}_1 \)? Is it possible?
Question 1  Definitions  (10 marks)

Give short definitions of the following:
  a) What is means by an ARFIMA(2,0.9,1) model?
  b) What is meant by an ADL(1,1) model?
  c) What is meant by a GARCH(1,1) in mean process?
  d) What is meant by a martingale process?

Question 2  Different Processes  (10 marks)

a) What is the expected value of an AR(1) process?
b) What are the theoretical autocorrelation and partial autocorrelation functions for the following processes:
   - White noise
   - Brownian motion
   - AR(2)
   - MA(3)
   - ARMA(2,1)

Question 3  Duality and Wold  (10 marks)

a) Show and discuss the duality between univariate MA and AR models, and for a VAR. Under which conditions can you go from one representation to the other?
b) Discuss the implications of duality for practical modeling, and add something about the Wald theorem and its implications for modeling and financial economic time series.

Question 4  Real Numbers  (10 marks)

Discuss the principles behind a so-called Granger causality tests
Consider the following information about the (log) share price index in South Africa:

<table>
<thead>
<tr>
<th>Year</th>
<th>LSPI</th>
<th>DLSPI</th>
<th>ACF-LSPI</th>
<th>PACF-LSPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4.5</td>
<td>5.0</td>
<td>5.5</td>
<td>6.0</td>
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</table>

In addition you have the following information:

**LSPI: ADF tests (T=111, Constant+Trend; 5%=-3.45 1%=-4.04)**

<table>
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<tr>
<th>D-lag</th>
<th>t-adf</th>
<th>beta</th>
<th>Y_1</th>
<th>sigma</th>
<th>t-DY_lag</th>
<th>t-prob</th>
<th>AIC</th>
<th>F-prob</th>
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<tbody>
<tr>
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**LSPI: ADF tests (T=111, Constant; 5%=-2.89 1%=-3.49)**

<table>
<thead>
<tr>
<th>D-lag</th>
<th>t-adf</th>
<th>beta</th>
<th>Y_1</th>
<th>sigma</th>
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<th>t-prob</th>
<th>AIC</th>
<th>F-prob</th>
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</tbody>
</table>

*Information to question 4 on next page.*
Question 5  (Use the information given above)  (10 marks)

a) Judging from the graphs, what type of process would be suitable for modeling the series? Motivate your answer as extensively as you can.

b) Is the series a random walk? Is the series integrated of order one? Motivate your answer.

c) Describe how the test for the order of integration is set up?

Question 6  Error correction and cointegration  (10 Marks)

a) What is the long-run solution to the following equation?

\[ y_t = 0.7 + 0.55y_{t-1} + 0.35y_{t-2} + 0.2x_{t-1} + 0.1x_{t-2} + 0.5z_{t-1} - 0.3z_{t-2} + \varepsilon_t \]

b) Rewrite the system as a general VAR model.

c) Discuss how you can use Johansen's method to test for cointegration, find out about cointegration among the variables. How many cointegrating vectors can you find? If \( y_t \) and \( x_t \) are two financial assets what conclusions can you draw from finding cointegration?

Question 1  10 marks

a. What is meant by a white noise process?

b. Find the expected value and variance of an AR(1) process.

c. Now we estimate an AR(2) model of some returns data
\[ y_t = 0.803y_{t-1} + 0.682y_{t-2} + \varepsilon_t \]

where \( \varepsilon_t \) is an assumed white noise process. How can you check the estimated model for stationarity?

**Question 2**

Consider the following autocorrelation and partial autocorrelation coefficients using 500 observations for a weakly stationary series:

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<tr>
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<th>ACF</th>
<th>PACF</th>
</tr>
</thead>
<tbody>
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</tr>
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</table>

**a.** Determine which, if any, of the ACF and PACF coefficients are significant at the 5% level.

**b.** Use both the Box-Pierce and Ljung-Box statistics to test the joint null hypothesis that the autocorrelation coefficients are jointly zero.

**c.** What process would you tentatively suggest could represent the most appropriate model for this series?

**d.** How could you estimate the model you suggest in part (c)?
Question 3  

10 marks

a) What are the characteristics of a variable integrated of order one, compared with a stationary variable? How can you test for the order of integration.

b) Assume that \( x_t \) is a typical economic time series, like the stock index, and \( y_t \) is GDP. Explain how you should go about testing if a long run relationship between \( y_t \) and \( x_t \) exists? In other words, is there a long run stable relation between stock prices and GDP?

c) Discuss briefly how you might test if GDP is causing stock prices to rise.

Question 4  

10 marks

a) Rewrite the following VAR as a Vector Error Correction Model,

\[
y_t = \pi_{11} y_{t-1} + \pi_{12} y_{t-2} + \pi_{13} x_{t-1} + \pi_{14} x_{t-2} + \epsilon_{1,t}
\]
\[
x_t = \pi_{21} y_{t-1} + \pi_{22} y_{t-2} + \pi_{23} x_{t-1} + \pi_{24} x_{t-2} + \epsilon_{2,t}
\]

b) Discuss some implication of cointegration in this model.

Question 5  

10 marks

a) What is meant by weak stationarity, as opposed to strong stationarity?

b) Why is important (briefly) to know whether a series is stationary or non-stationary?

c) The lag polynomial of an AR model tells us about the path of the time series process after a shock. Explain what we can read out regarding the time path of a variable from the lag polynomial?

Question 6  

10 marks

The principle of duality is central in time series modeling,

a) What does it mean? (Show the math)

b) When is it applicable on an AR process on an MA process?

c) What is/are the practical implication(s) of duality (if any)?
Give short definitions of the following:

- e) What means by an ARFIMA(2,0.9,1) model?
- f) What meant by an ADL(1,1) model?
- g) What meant by a GARCH(1,1) in mean process?
- h) What meant by a martingale process?

**Question 2  Different Processes** (10 marks)

a) What is the expected value of an AR(1) process?
b) What are the theoretical autocorrelation and partial autocorrelation functions for the following processes:
   - White noise
   - Brownian motion
   - AR(2)
   - MA(3)
   - ARMA(2,1)

**Question 3  Duality and Wold** (10 marks)

c) Show and discuss the duality between univariate MA and AR models, and for a VAR. Under which conditions can you go from one representation to the other?
d) Discuss the implications of duality for practical modeling, and add something about the Wald theorem and its implications for modeling and financial economic time series.

**Question 4  Real Numbers** (10 marks)

d) Discuss the principles behind a so-called Granger causality tests
Question 6  Error correction and cointegration  

(10 Marks)

a) What is the long-run solution to the following equation?

\[ y_t = 0.7 + 0.55y_{t-1} + 0.35y_{t-2} + 0.2x_{t-1} + 0.1x_{t-2} + 0.5 z_{t-1} - 0.3 z_{t-2} + e_t \]

b) Rewrite the system as a general VAR model.

c) Discuss how you can use Johansen's method to test for cointegration, find out about cointegration among the variables. How many cointegrating vectors can you find? If \( y_t \) and \( x_t \) are two financial assets what conclusions can you draw from finding cointegration?