OPTIMAL CONTROL FOR HYDRAULIC SYSTEM WITH SEPARATE METER-IN AND SEPARATE METER-OUT

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Abstract

Individual meter-in and meter-out proportional control of hydraulic cylinders allow more flexible strategies how to handle a payload while optimising certain aspects of the motion. An optimal control approach based on the Hamiltonian is used to plan a motion path for a separate meter-in and separate meter-out system. One goal is to avoid oscillations at the end of the motion in the sense of active vibration damping. With the mathematical model presented, it is possible to include the cylinder pressures into the optimisation goals. Not only the the final values for position and speed of the load can be acquired, but also desired values for the pressures at the end of the motion. Maintaining a minimum pressure level is important in order to avoid cavitation and to keep the stiffness of the system. The course of the pressures follows from the design of the motion and will reach the desired values in an open loop control. In order to estimate the computing effort, the algorithm was tested on three platforms. One of those is an ARM-based hardware that represents the computing power of modern embedded systems, to demonstrate that an implementation on such system is possible. Several motion experiments were carried out in simulation to study the behaviour and to discuss application issues.
Keywords: Hydraulic actuator, optimal control, separate meter-in/separate meter-out

1 Introduction

In standard hydraulic systems with proportional valves speed is controlled by opening or closing two orifices on a single spool. Careful design of load-sensing and pressure compensation is necessary for a good performance [1]. Introduction of a second valve in separate meter-in/separate meter-out controlled systems (SMISMO) gives possibility to control one additional variable, usually back actuator pressure independent from speed of the actuator [2] [3] [4]. This possibility to control the back pressure is especially useful when moving high inertia loads fast, since in this case, stopping the load using only one valve without having oscillation is extremely difficult [5] [6]. In those cases SMISMO has the potential to make the hydraulic system stiffer, to increase the back pressure, and with this to obtain less vibrations [7] [8].

Motion of payloads needs energy, causes oscillations and wears mechanical systems. For this reasons, optimisation methods are of increasing importance to minimise or maximise some physical property. In vibration damping, optimisation is applied in order to have a high energy dissipation [9]. Or, the motion path of a manipulator is planned so, that stress is equally distributed over the links [10]. Effort is done to integrate optimal control in the function of a hydraulic valve [11]. Since hydraulic systems are highly nonlinear, it is also evident to consider such behaviour for design of optimal controllers [12].

Several approaches are used for optimisation. A common technique is input shaping, that forms the signal sent to the servo-valve in order to suppress oscillations. Such shaping filters can be applied to the input of a drive system [13], to the input of a control circuit [14], or within a loop between controller and system [15]. The latter presents an application in a hydraulic system. Another approach is the design of linear-quadratic regulators (LQR) based on the solution of the Matrix Ricatti differential equation [11] [12]. Optimisation based on the Hamiltonian (e.g. in [9]) is used for the actual work.

Recent development of electronic hardware and software allows an increasing integration of complex algorithms in industrial control. Modern hydraulic valves offer simple integration of separate meter-in and separate meter-out control with various operation modes [16]. This makes optimal control techniques more appealing for practical application in servo-hydraulic systems.

In this work a Hamiltonian-based approach is presented, which offers movement of the high inertia loads without oscillations and open-loop control of speed and position of the cylinder. Additionally, the pressures in the actuator are included in the design. Firstly, a mathematical model of a servo-hydraulic drive is derived in state-space representation as a linear time-invariant (LTI) system. Then the optimisation goal is described in the form of a functional. Using the Hamiltonian, the differential equation system, which delivers the optimal control, is found. The numerical solution requires finding the initial conditions first, before the system can be solved. This is demonstrated with an example. Then implementation and numerical effort are considered in order to estimate calculation times. Finally, in section six, some case studies are shown to demonstrate the correctness of the approach and to allow discussion.

2 Model of the System

2.1 Mechanical System

The mechanical system consists of a big mass moved with a lever rotating around the point O (fig. 1). For modelling purposes the mass of the system is concentrated in a single point with a mass of M, the lever of the length L is regarded to be massless. A hydraulic cylinder applies the force F to the small lever of the length l. For small motions around the actual point the effective mass is

$$m = \left(\frac{L}{l}\right)^2 M. \tag{1}$$

The motion is carried out horizontally, hence no force of gravity is regarded.

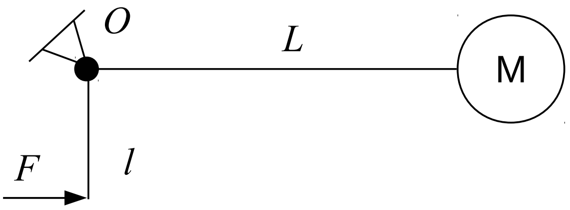

Figure 1: Sketch of the mechanical system

2.2 Hydraulic System

The main parts of the hydraulic system in fig. 2 are a double acting, single rod cylinder and two proportional valves that allow individual control of the oil flows q_A and q_B. Auxiliary components, like load-sensing and pressure compensation are not displayed in the figure.

2.3 Mathematical Model

If the position of the mass m in fig. 2 is designated as x_1, the motion equation follows with

$$m\ddot{x}_1(t) = p_A(t)\cdot A_A - p_B(t)\cdot A_B - b\cdot \dot{x}_1(t). \tag{2}$$

In a system with hydraulic actuators, friction consists of several components. The motion of the payload contributes, also the motion of the steel structure. Oil flow over an orifice depends on the pressure and hence causes additional damping.

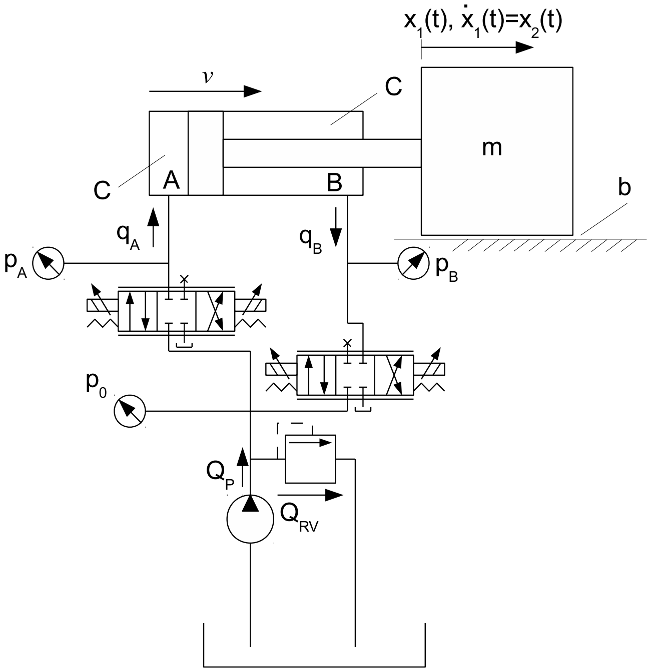

Figure 2: Hydraulic system

Finally, the sealing in the cylinder offers a highly nonlinear friction behaviour. A comprehensive mathematical model of friction includes Coulomb and viscous friction, stiction and the Stribeck effect [17]. In the actual machine, the main contribution is coming from a rotating load that performs a cutting process. It is assumed to be viscous with a coefficient of b in the dynamic model eq. (2).

The motion is controlled with two independent valves. The electronic control provides several operation modes [16], e.g. to establish constant oil flows automatically corrected with the help of integrated pressure sensors. This allows direct control of the oil flows, hence we use the oil flows q_A and q_B as inputs, which change the pressures in the cylinder

$$\begin{aligned} \dot{p}_A(t) &= \tfrac{1}{C_A}\left(q_A(t) - A_A\cdot \dot{x}_1(t)\right) \\ \dot{p}_B(t) &= \tfrac{1}{C_B}\left(q_B(t) + A_B\cdot \dot{x}_1(t)\right). \end{aligned} \tag{3}$$

The hydraulic capacity can be calculated for a volume V with

$$C = \frac{V}{E}, \tag{4}$$

where E is the bulk modulus of the fluid. The hydraulic capacities C_A and C_B depend on the volumes of the chambers and hence on the position of the piston, which results in a nonlinear system. The assumption of a constant capacity means, that the considerations are valid only for small position changes. The compliance of the tubes between valves and cylinder also contribute to the capacity. For simplification, we assume both chambers to have the same values, hence $C_A = C_B = C$.

If a new variable x_2 is introduced for the speed of the system

$$\dot{x}_1 = x_2, \tag{5}$$

the system can be written in its state-space formulation

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{p}_A \\ \dot{p}_B \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b}{m} & \frac{A_A}{m} & -\frac{A_B}{m} \\ 0 & -\frac{A_A}{C} & 0 & 0 \\ 0 & \frac{A_B}{C} & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ p_A \\ p_B \end{pmatrix} + \\ + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{C} & 0 \\ 0 & \frac{1}{C} \end{pmatrix} \cdot \begin{pmatrix} q_A \\ q_B \end{pmatrix}. \tag{6}$$

3 Optimal Control

The state-space representation can be used to design a digital controller according to methods found in [18]. Here an optimal control approach is chosen based on variational calculus using the Hamiltonian [19].

If the quality criterion is selected to be

$$J(x,q) = \frac{1}{2}\int \left(\mathrm{x^T Q x} + \mathrm{q^T R q}\right) \cdot \mathrm{d}t, \tag{7}$$

the matrix Q rates the state variables during the motion, and the matrix R regards the input variables, which are the oil flows in and out from the cylinder chambers. The matrix R rates the sum of the squares of the flows q_a and q_b during the motion.

Usually the functional is used to minimise energy consumption, but in the case of hydraulic systems, the power delivered by the supply is not to be described with linear functions. Anyway, it is a good idea to minimise the square of the oil flows, since it is related to the opening of the valve orifices. In the similar way, the square of the motion variables (rated with the matrix Q) minimises elongations and oscillations in the system.

Now the Hamiltonian [19] can be formulated as

$$\mathrm{H} = -\frac{1}{2}\left(\mathrm{x^T Q x} + \mathrm{q^T R q}\right) + \psi^T\left(\mathrm{Ax} + \mathrm{Bq}\right). \tag{8}$$

Moving on the optimal path,

$$\frac{\partial \mathrm{H}}{\partial \mathrm{q}} = 0 \tag{9}$$

yields the control law

$$-2\,\mathrm{Rq} + \mathrm{B}^T\psi = 0 \tag{10}$$

and finally

$$\mathrm{q} = \frac{1}{2}\mathrm{R}^{-1}\mathrm{B}^T\psi. \tag{11}$$

The adjunct system ψ follows from the canonical equations with

$$\dot{\psi} = -\frac{\partial \mathrm{H}}{\partial \mathrm{x}} = 2\mathrm{Qx} - \mathrm{A}^T\psi \tag{12}$$

and then

$$\dot{\mathrm{x}} = -\frac{\partial \mathrm{H}}{\partial \psi} = \mathrm{Ax} + \mathrm{Bq} = \mathrm{Ax} + \frac{1}{2}\mathrm{BR}^{-1}\mathrm{B}^T\psi. \tag{13}$$

Putting Equations 12 and 13 together delivers the *canonical system*

$$\begin{pmatrix} \dot{\mathrm{x}} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \mathrm{A} & \frac{1}{2}\mathrm{BR}^{-1}\mathrm{B}^T \\ \mathrm{Q} & -\mathrm{A}^T \end{pmatrix} \cdot \begin{pmatrix} \mathrm{x} \\ \psi \end{pmatrix} \tag{14}$$

The numerical solution of this system yields the controlled variable q, which is a vector of the input flows q_A and q_B, from the adjunct system ψ in eq. (11).

4 Numerical Solution

4.1 Initial Conditions

For solving the canonical system in eq. (14) the initial conditions of

$$\mathrm{x}(0) = \begin{pmatrix} x_1(0) & x_2(0) & p_A(0) & p_B(0) \end{pmatrix}^T \tag{15}$$

with a number of $n = 4$ are known. The system is eight by eight, the initial conditions

$$\psi(0) = \begin{pmatrix} \psi_1(0) & \psi_2(0) & \psi_3(0) & \psi_4(0) \end{pmatrix}^T \tag{16}$$

are still missing. With the final conditions

$$\mathrm{x}(t_f) = \begin{pmatrix} x_1(t_f) & x_2(t_f) & p_A(t_f) & p_B(t_f) \end{pmatrix}^T \tag{17}$$

the initial condition for ψ can be found. With the abbreviation

$$\mathrm{M} = \begin{pmatrix} \mathrm{A} & \frac{1}{2}\mathrm{BR}^{-1}\mathrm{B}^T \\ \mathrm{Q} & -\mathrm{A}^T \end{pmatrix} \tag{18}$$

the canonical system eq.(14) turns to

$$\begin{pmatrix} \dot{\mathrm{x}} \\ \dot{\psi} \end{pmatrix} = \mathrm{M} \cdot \begin{pmatrix} \mathrm{x} \\ \psi \end{pmatrix}. \tag{19}$$

This system can be solved for the final target time t_f with the help of the exponential matrix and gives

$$\begin{pmatrix} \mathrm{x}(t_f) \\ \psi(t_f) \end{pmatrix} = e^{\mathrm{M}\cdot t_f} \cdot \begin{pmatrix} \mathrm{x}(0) \\ \psi(0) \end{pmatrix} \tag{20}$$

After partitioning the exponential matrix

$$e^{\mathrm{M}\cdot t_f} = \begin{pmatrix} \mathrm{S}_{11} & \mathrm{S}_{12} \\ \mathrm{S}_{21} & \mathrm{S}_{22} \end{pmatrix} \tag{21}$$

the initial conditions can be found with

$$\psi(0) = \mathrm{S}_{12}^{-1} \cdot \left(\mathrm{x}(t_f) - \mathrm{S}_{11} \cdot \mathrm{x}(0)\right). \tag{22}$$

4.2 Solution and Control

Now the canonical system eq. (14) can be solved as an standard initial value problem (IVP). Then, with the help of the control law eq. (11), the oil flows q_A and q_B follow, which should be used as controlled variables for the hydraulic system. The numerical solution of eq. (14) additionally yields a model of the physical system. It also contains the pressures as state variables. This information can be compared with sensor values of the real system and help to add some feedback correction for the nonlinear characteristics of the valves.

4.3 Application Example

The system parameters are cylinder areas of $A_A = 0.06\,\mathrm{m}^2$ and $A_B = 0.04\,\mathrm{m}^2$, a reduced mass of $m = 300000\,\mathrm{kg}$, the hydraulic capacity of one oil chamber $C = 10^{-11}\,\mathrm{m}^3\mathrm{Pa}^{-1}$. The viscous friction b was assumed to be zero to ensure, that all oscillation damping is done by the control algorithm.

Now the quality functional eq. (7) is established with the matrix

$$\mathrm{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{23}$$

which rates the squared size of the oils flows. Here q_A and q_B have the same weight.

The matrix Q describes the contribution of state variables to the functional eq. (7), which should be minimised. With

$$\mathrm{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{24}$$

the speed variable x_2 is rated. All state variables (the position x_1, the speed x_2, and the pressures p_A and p_B) have defined initial and end values. Taking the speed into the matrix Q leads to the damping of oscillations.

The initial parameters are a position $x_1(0) = 0\,\mathrm{m}$, an initial speed $x_2(0) = 0\,\mathrm{ms}^{-1}$ and the pressure $p_A(0) = 10\,\mathrm{bar}$. The dynamic and static force is initially zero, from this follows the initial value for p_B.

The target time for the motion is $t_f = 0.8\,\mathrm{s}$, the position should be $x_f = 0.01\,\mathrm{m}$. The pressure in chamber A at the end of the motion should be $p_A(t_f) = 50\,\mathrm{bar}$. The pressure in B follows from p_A, when no external load is present at the end position.

The simulation result in fig. 3 shows that the motion targets are accomplished. The load stands still, no force is applied, hence no oscillation remains, and the cylinder chambers have the desired pressure levels.

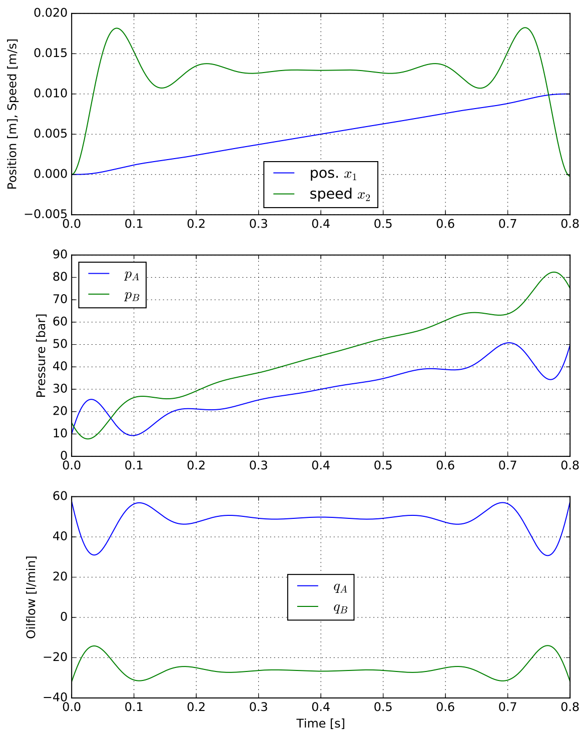

Figure 3: Variables during the motion

5 Implementation

5.1 Computing Procedure

The procedure to plan and to control a motion follows three steps.

5.1.1 Model Initialisation

At first eq. (18), which is the dynamic matrix of the canonical system, must be established. The matrix depends on the physical parameters of the system and will not change during the motion. When the load changes, for example, the matrix will be affected and the step has to be repeated.

5.1.2 Processing Boundary Conditions

The next task in planning the motion is to process the desired target values (maybe from operator input), the initial conditions (usually from sensors) and the final time. The planned final time of the motion t_f is required to find the exponential matrix for eq. (20), to partition it and to invert the partition S_{12} in eq. (22). With the boundary conditions of the state variables x, eq. (22) yields the initial values for the adjunct system ψ.

5.1.3 IVP solution

Now the initial value problem (IVP) eq. (14) can be solved. It delivers the state variables x, which is a simulation model of the system. Furthermore, the IVP solution outputs ψ. The oils flows q_A and q_B follow with the help of eq. (11) and can be output as controlled variables.

There are two possibilities to solve the IVP.

1. Solution before the start of the motion: The calculation is done after processing the boundary conditions, but the output of the controlled variables starts afterwards together with the motion cycle. For this case, a fixed-step-size ODE solver from a library can be used.

2. The solution is done during the motion. On each sample time instant values for the outputs are computed, which are forwarded to the valve control. This computing mode needs fixed-step-size ODE solvers that can return a new single value after each call. In the examples in this work, a simple forward Euler method was implemented.

5.2 Hardware and Software

Electronic controllers for industrial control must be robust and dependable. In order to achieve this also for the programming software, standard languages were developed early [20] and are still used today. They are not suitable to program complex systems, like they are demanded today [21], consequently modern industrial controls allow to implement software modules written in different languages. The optimisation algorithms described in this work could be written directly in C++, using e.g. the ODEINT library to solve the IVP. To avoid a general purpose language, cross-development can be used to generate code that can run on a real-time system. For example, Matlab®/Simulink® provides a suite for this purpose. The C code generated with such tools can be compiled in the environment of the programmable controller und be integrated in the automation system.

A second approach to integrate a program for complex algorithms with a machine control is to utilise modern communication, preferably UDP from the TCP/IP suite. The optimal control can run on an industrial PC that communicates with the PLC.

5.3 Computing Times

For the task to implement the actual algorithm on a computer or an embedded system, it is important to know about the numerical effort. In order to give a feeling for the required calculation, three time measurements were done. The first was a PC with Windows, i3-2120 CPU @ 3.3 GHz (PC/W in tab 1), the second a PC with Linux, i5-4210U CPU @ 1.70 GHz (PC/L in tab 1), and the third, closer to an embedded system, a Raspberry Pi 3B with Raspbian Linux, ARM Cortex-A53 @ 1.2 GHz (ARM in tab 1).

The language for the test was Python with the advantage, that the same code runs on all three platforms without any modification. The results are shown in tab 1.

Table 1: Computing times.

Step	PC/W	PC/L	ARM	Unit
Initialisation	24	0.33	6	[ms]
Boundary conditions	30	13	210	[ms]
IVP solution	0.6	0.3	6.4	[s]
IVP solution per step	36	19	410	[μs]

The initialisation is just the assembling of the matrices, processing boundary conditions requires mainly calculating the exponential of an 8 by 8 matrix and the inversion of a 4 by 4 matrix. The solution of the IVP is done with Forward Euler method in 16 000 steps.

5.4 Practical Aspects

The numerical solution of the canonical system eq. (14) delivers the adjunct system ψ, from which the vector q (containing the controlled variables q_A and q_B) follows with eq. (11). If the voltage signals fed to the servo valve represents the opening of the orifices, the resulting oil flows are not proportional to the control output. Hydraulic or electronic pressure compensation improves the situation.

With additional pressure sensors, a control loop can be established to maintain the desired oil flows, e.g. like in [16].

In [22] the following approach was presented and verified. Valve opening is predicted for one flow direction with

$$\begin{aligned} y_A &= y_{\max} \cdot \frac{Q}{Q_N} \cdot \frac{\sqrt{\Delta p_N}}{\sqrt{p_P - p_A}} \\ y_B &= y_{\max} \cdot \frac{Q}{Q_N} \cdot \frac{\sqrt{\Delta p_N}}{\sqrt{p_B - p_T}}, \end{aligned} \tag{25}$$

where y_A and y_B are the signals from the controller (here the desired value for flow, q_A or q_B), y_{max} is the limit for the input, Q represents the desired flow value. Q_N and Δp_N are the nominal parameters of the orifice, p_P is the pressure on the pump side, and p_T on the tank side. When a pressure sensor fails, the pressure terms in eq. (25) can be omitted. Further fail-safe features in different operating situations can be found in [22].

The solution of the canonical system eq. (14) also outputs the state variables of the mechanical system. This is a simulation model, which delivers also predictions for the pressures in the cylinder. The information about the desired pressures p_A and p_B can be used to establish a multi-variate control for flow and pressure.

6 Simulation Results

6.1 Some Case Studies

Figure 3 shows that the motion ends with the desired load position and a speed of zero after the proposed time. Additionally, the cylinder pressures meet the demanded values.

High pressure levels increase the power loss, lower levels may cause cavitation and reduce the stiffness of the oil volume. In the motion example in fig. 3 pressures do not fall under a value of 10 bar. If the motion is turned into the other direction (fig. 4), cavitation occurs after the start. In this case, the system gets nonlinear, and the simulation model is not valid anymore. With the initial lowest pressure of 50 bar, the problem does not occur (fig. 5).

Consequently it is better to keep the desired pressure at 50 bar for this example. The forward motions will look like in fig. 6, where the lowest pressure occurs in the breaking phase.

The motion in fig. 7 is too fast to damp the oscillation properly without causing cavitation.

Figure 8 displays the results of a longer or slower motion. The solution gives a constant value for the speed after the oscillation is vanished. It shows that the problem could be split into a start phase and a stop phase. This approach can be used for a travel with constant speed, and the design of the path will be independent from the distance.

When the travelling time is increased while the number of steps is held constant, the step width has to grow. Figure 9 shows that the solution of the IVP, eq. (14), does not meet the end conditions exactly. This is a problem with stiff numerical systems. It turns out, that many solvers have the problem with the actual example. One critical point is the computation of the exponential matrix in eq. (20) [23]. Current work is

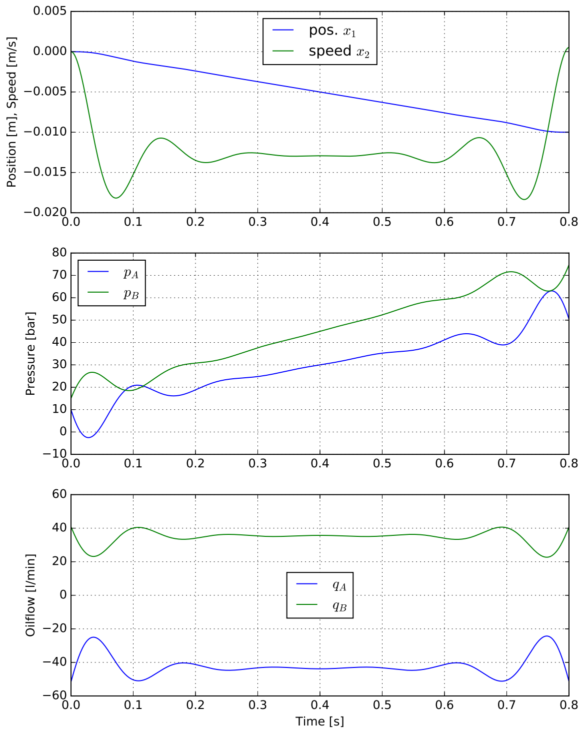

Figure 4: Reverse motion with too low initial pressures

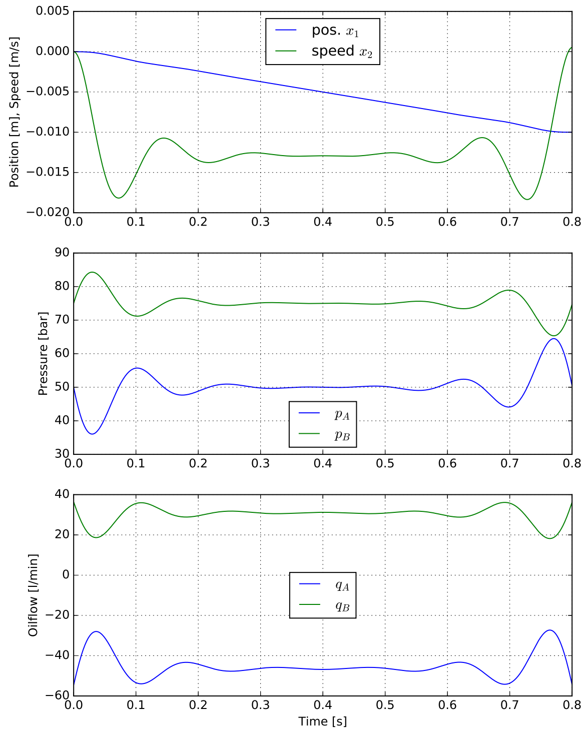

Figure 5: Reverse motion

done to work on a solving method that always meets the final conditions exactly [24].

6.2 Comparison With Start/Stop Motion Control

A system is taken for a simulation with Simulink®, which has the same properties as the system before, but with a damping of $b = 5 \cdot 10^6$. For a motion a constant input signal was generated with a ramp-up and ramp-down of 0.1 s. The input level was adjusted experimentally so, that the same travelling distance is reached. The oil flow q_A is directly proportional to the signal input, q_B is determined by an orifice, that opens synchronously with the input signal. The size of the orifice was chosen experimentally such that the desired pressure levels at the final position are obtained. In the real system, this is a standard operating mode provided by the electronic valve control [16]. Additionally meter-in and meter-out valves have a first-order time delay of 30 ms. A positioning experiment is done and compared with the results of the proposed method. In both cases the valves are closed after the target time of 0.8 s.

Figure 10 shows the positions over time, fig. 11 the speeds, fig. 12 the pressures in the cylinders, and fig. 13 the oil flows. It can be seen from these diagrams that the proposed procedure brings the load to the end position without leaving oscillations after the end time. The time delays of the spools with a value of 30 ms were not included in the design of the optimal control, since the numerical solvers used could not find a solution for this problem. A new approach [24] will make this possible. The additional delay is the reason, why the valves do not close immediately at the end time. The consequence is a short delay of the stopping time instance, but still the target conditions are met without oscillation, which is an advantage against the start/stop control.

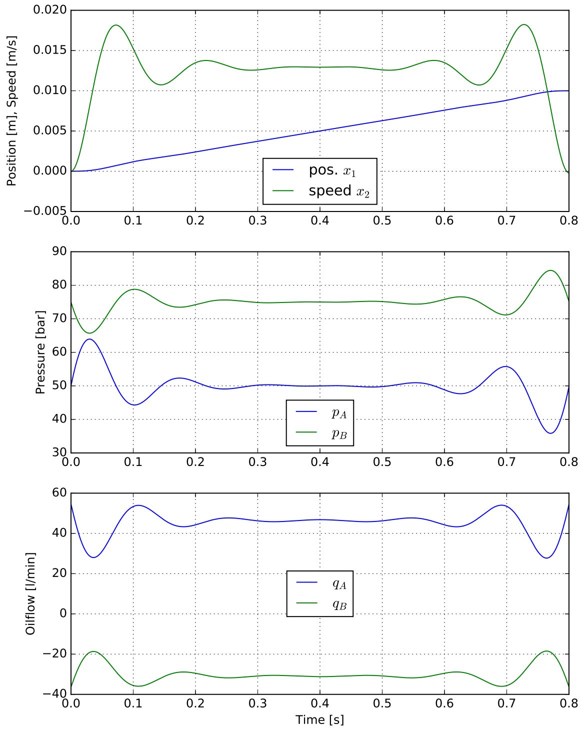

Figure 6: Forward motion starting at $p_A = 50$ bar

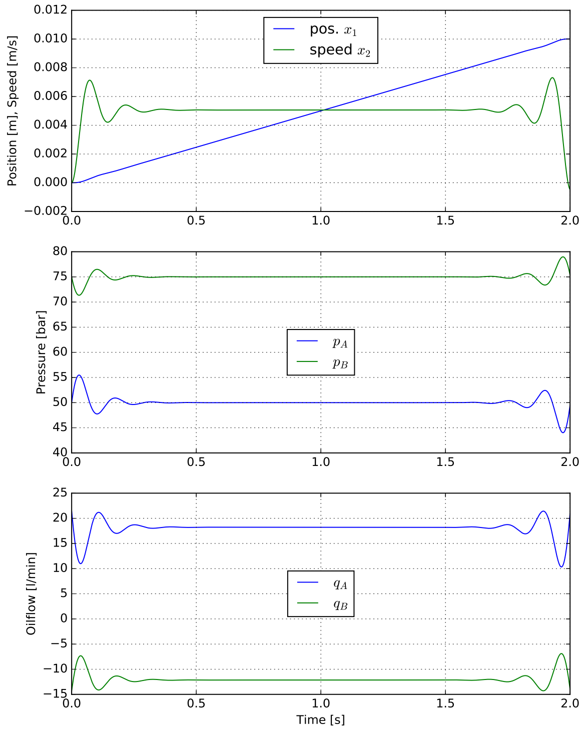

Figure 8: Long-time motion with quasi-stationary travel

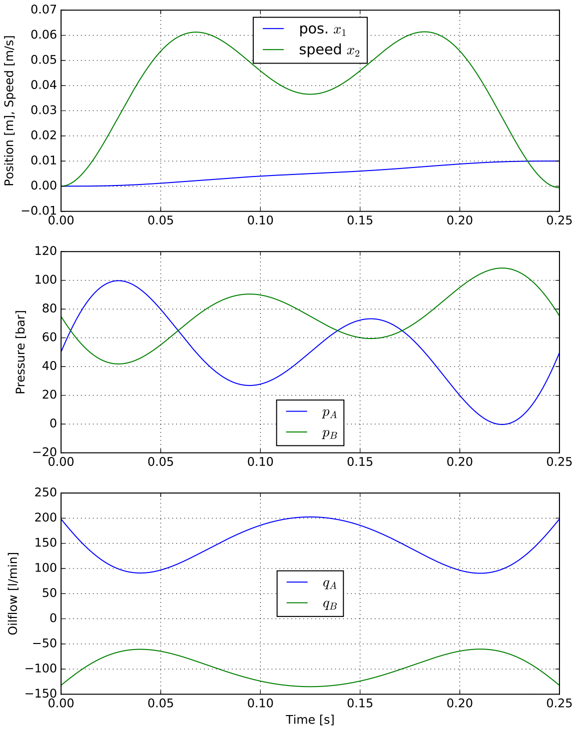

Figure 7: Motion in shorter time, appearently too fast

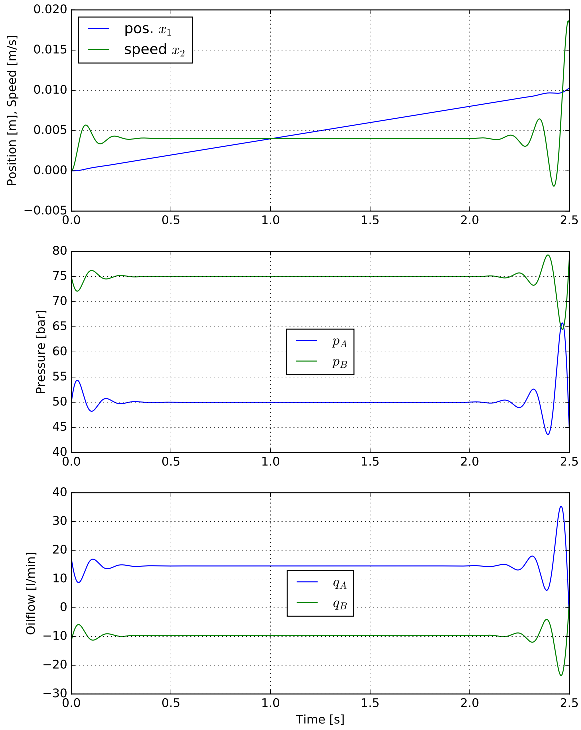

Figure 9: Demonstration of numerical problems

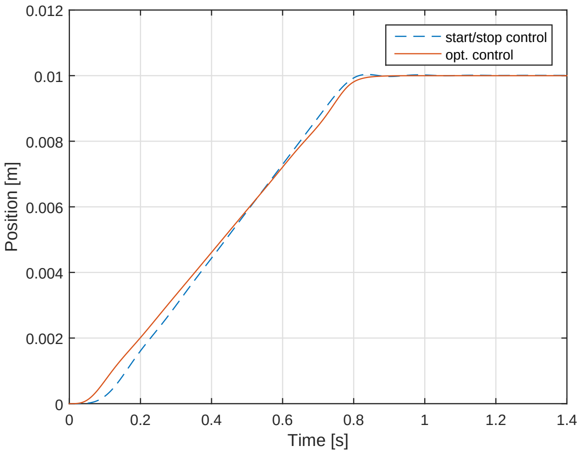

Figure 10: Positions start/stop vs. optimal control

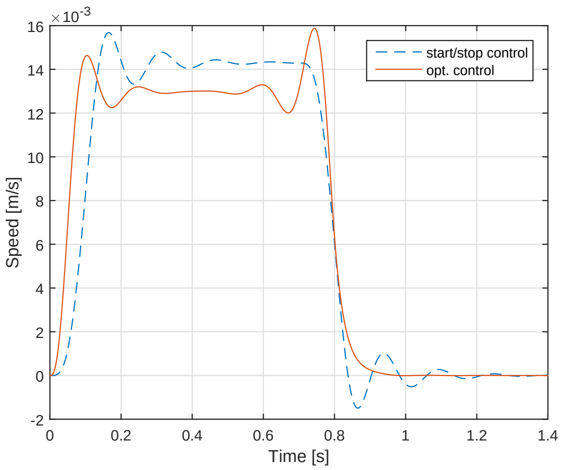

Figure 11: Speeds start/stop vs. optimal control

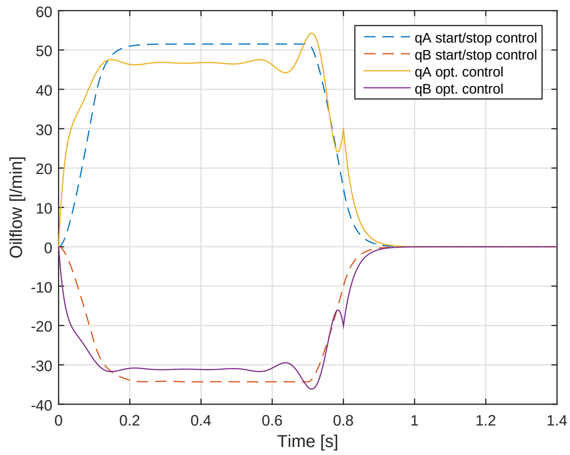

Figure 13: Oils flows start/stop vs. optimal control

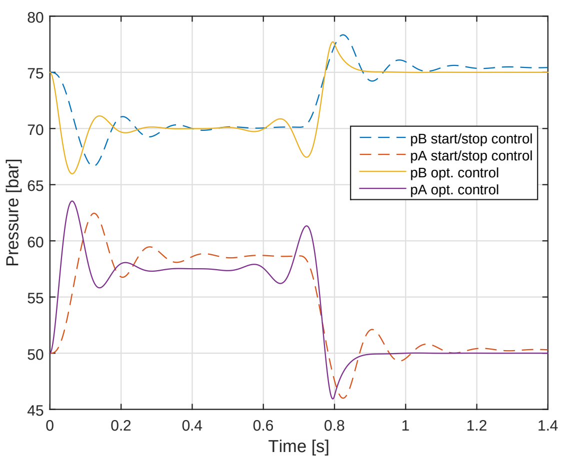

Figure 12: Pressures start/stop vs. optimal control

7 Conclusions

The proposed method of designing a motion for a SMISMO-controlled hydraulic system allows suppressing oscillations at the end of the motion. In a separate-meter-in/separate-meter-out system it is possible to include the pressure levels in the cylinder into the motion design procedure, which means, desired values are met after the motion is finished. This is important to keep the pressures low for energy efficiency and also to avoid cavitation. The computing times are short enough to be applied for motion planning in industrial automation or even in embedded systems.

Some case studies with a simulation model with different speeds in both directions show that the results are practically reasonable. The outputs of the calculation are desired flow values. To establish constant flow in the physical system, hydraulic or electronic pressure compensation is required. This can be provided by modern valves with integrated electronics. As an alternative, a proposal to control a proportional valve with an external linarisation model was given.

It turns out that ODE solvers have problems, when the planned time exceeds a certain length. Even dedicated stiff solvers can fail. A simple Euler method with small step size turned out to work well. This kind of numerical problems is a topic of ongoing research.

Nomenclature

Designation	Denotation	Unit
x_1	Position of the load	m
x_2	Speed of the load	m/s
t_f	Final target time of the motion	s
p_A	Pressure piston side (chamber A)	Pa
p_B	Pressure rod side (chamber B)	Pa
p_P	Pressure valve input on pump side	Pa
p_T	Pressure valve output on tank side	Pa
q_A	Flow into chamber A	m^3/s
q_B	Flow into chamber B	m^3/s
Q	Oil flow through a valve	m^3/s
Q_N	Nominal oil flow of a valve m^3/s	
p_N	Nominal pressure for Q_N	Pa
y	Control input of valve	V
y_{max}	Max. control input of valve	V
m	Reduced mass	kg
A_A, A_B	Areas	m^2
b	Viscous friction coefficient	Ns/m
C	Hydraulic capacity	m^3/Pa
E	Bulk modulus	N/m^2
V	Volume	m^3

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